ABSTRACTS OF ARTICLES DEPOSITED AT VINITI*

NATURAL CONVECTIVE HEAT TRANSFER AND RESISTANCE FOR A HEATED SPHERICAL BODY WITH A LAMINAR BOUNDARY LAYER

A. A. Gusakov and G. A. Kolykhalov

UDC 536.24:533.6

Stationary natural convection is considered for a uniformly heated solid sphere at high Prandtl-Grashof numbers. No detachment occurs in the flow around the sphere.

The solution is based on the equations of continuity, motion, and energy conservation for a compressible medium in the boundary-layer approximation. The density and viscosity are taken as temperature dependent, with the density determined on the assumption that the pressure difference within the boundary layer is much less than the pressure outside the boundary layer, or the viscosity is linearly dependent on temperature. The successive approximation solution is based on a layer of finite thickness (Shvets's method) and is restricted to the second approximation. A dimensionless relationship has been derived for the heat-transfer factor, which agrees well with experiment.

The theoretical value for the natural convection resistance coefficient has been determined on the assumption that there is an additional (lifting) force apart from the frictional force and the pressure difference, which is due to the density difference between the media outside and within the boundary layer.

Experiments on natural convective resistance agree well with the theory; these measurements were made at the Prandtl—Grashof numbers between $3 \cdot 10^4$ and $3 \cdot 10^{-5}$.

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Original article submitted April 11, 1973.

VELOCITY AND TEMPERATURE DISTRIBUTIONS OF A TURBULENT AIR FLOW IN A HORIZONTAL PIPE UNDER THE INFLUENCE OF THERMOGRAVITATIONAL FORCES

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UDC 532.517.4

The distributions of the average velocity and temperature in a turbulent air flow in a horizontal pipe are investigated experimentally.

The experimental section consists of a circular pipe with an inside diameter of 144 mm. The length of the unheated entrant zone of the pipe is 20 diameters, and the length of the heated zone is 50 diameters. Heating is realized by the direct passage of an alternating current through the pipe wall. The wall temperatures are measured along the tube with 90 thermocouples. The temperature distribution in the flow cross section is measured by means of a thermocouple probe, and the velocity profile with a Pitot tube and hot-wire anemometer. The results of preliminary measurements of the local heat transfer and the velocity and temperature fields free of the influence of thermogravitational forces are consistent with the existing published data.

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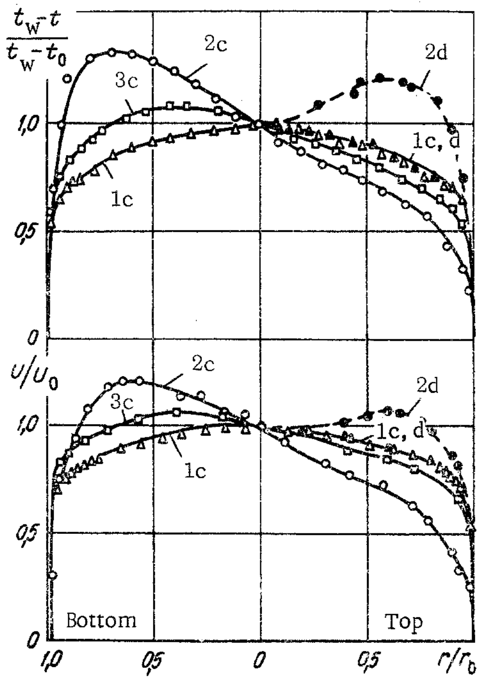

Fig. 1

The velocity and temperature fields subject to the influence of thermogravitational forces are measured in the ranges of Reynolds and Grashof numbers

$$\mathrm{Re} = \frac{\bar{u}d}{\nu} = (1,3-5,2)\cdot 10^4; \quad \mathrm{Gr} = \frac{g\beta q_c d^4}{\lambda \nu^2} = (0.2-2)\cdot 10^9.$$

The results of measurements of the average velocities and temperatures in the vertical and horizontal diametric planes far from the start of heating are summarized in tables and figures, one of which is included in the abstract (Fig. 1).

It is evident from the figure that for Re = $5.2 \cdot 10^4$ and Gr = $0.2 \cdot 10^9$ (curve 1) the velocity and temperature profiles in the vertical (1c) and horizontal (1d) planes coincide and are symmetrical, indicating the noninfluence of thermogravitational forces. On the other hand, for Re = $1.3 \cdot 10^4$ and the same value of Gr (curve 2) an appreciable asymmetry is noted in the velocity and temperature profiles in the vertical diametric plane (2c). With an increase in Gr (curve 3), however, even for Re = $5 \cdot 10^4$ the influence of the thermogravitational forces on the velocity and temperature profiles is significant.

For Re = $1.3 \cdot 10^4$ (curve 2) the velocity and temperature distributions have maxima in the horizontal diametric plane. The pattern of the velocity and temperature distributions in this regime is qualitatively reminiscent of the distribution of the relative velocity and temperature values in the case of viscogravitational flow in horizontal pipes. The maxima in the velocity and temperature distributions with respect to the horizontal diameter indicate the presence of secondary currents in the given case.

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AN APPROXIMATE ANALYTIC SOLUTION OF THE TWO-DIMENSIONAL RADIATIVE HEAT-TRANSFER PROBLEM

B. I. Medvedev, S. I. Gertsyk, and A. G. Zen'kovskii UDC 536.3

We consider the two-dimensional problem of radiative heat transfer with sources in a stationary gray medium between two infinite parallel gray planes at specified temperatures.

By using the Green's function method exact formal expressions are obtained for the temperature distribution of the medium and the radiation fluxes.

It is shown rigorously that in the absence of heat sources and for a linear variation of the fourth powers of the boundary temperatures along the longitudinal coordinate the temperature distribution of the medium and the vertical radiation fluxes can be found from the solution of a one-dimensional problem, i.e., for boundary temperatures constant lengthwise.

For the practically important case of a smooth change of the boundary temperatures, when the characteristic distance over which the boundary temperatures change is not less than the distance between the planes bounding the system, approximate expressions are obtained for the temperature distribution in a layer and the radiant heat fluxes in the form of rapidly converging series. It follows from these expressions that for values of the longitudinal optical thickness τ of the order of a few transverse optical thicknesses τ_0 the re-emission of the medium smooths out the effect of the parts of the boundaries farther away than τ. Therefore, for smoothly varying boundary temperatures the one-dimensional approximation gives good results.

Results are presented of a numerical calculation for a point source when the temperature of the boundaries varies according to the law $\theta \sim \exp(-\tau)$ for $\tau_0 = 1$.

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NONSTATIONARY HEAT CONDUCTION IN A HEMISPHERE

S. I. Prokopets

UDC 536.24.02

The nonstationary temperature distribution in a hemispherical shell is a solution of the following boundary value problem:

$$\frac{\partial t}{\partial \tau} = a\left\{\frac{\partial^2 t}{\partial r^2} + \frac{2}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right)\right\}, \quad (1)$$

$$t(r, \theta, \tau) = t_0(r, \theta) \text{ for } \tau = 0, \quad (2)$$

$$t(r, \theta, \tau) = f(r, \tau) \text{ for } \theta = \frac{\pi}{2}, \quad (3)$$

$$|t(r, \theta, \tau)| < \infty \text{ for } \theta = 0, \quad (4)$$

$$\lambda\frac{\partial t}{\partial r} - \alpha[t - t_1(\theta, \tau)] = 0 \text{ for } r = R_1, \quad (5)$$

$$\lambda\frac{\partial t}{\partial r} + \alpha[t - t_2(\theta, \tau)] = 0 \text{ for } r = R_2. \quad (6)$$

We seek the solution of this problem in the form of a series in the eigenfunctions of the Sturm-Liouville problem in the variable r. The convergence of the series obtained is improved and the boundary conditions on θ are satisfied by the G. A. Grinberg method. Finally, the solution of the problem takes the form

$$t(r, \theta, \tau) = \frac{2}{\pi}[f(r, \tau) - h(r, \tau)]\theta + t_2(\theta, \tau) + \left(c + \frac{b}{r}\right)[t_1(\theta, \tau) - t_2(\theta, \tau)]$$
$$+ \sum_{n=0}^{\infty}\left\{\sum_{k=1}^{\infty} W_{nk}(\tau) N_{nk}^{-1} M_p(\gamma_{nk} r) - a_{nk}\right\} P_{2n+1}(\theta), \quad (7)$$

where the $M_p(\gamma_{nk} r)$ are the eigenfunctions of the problem — Bessel functions of the first kind of order $p = n + 1/2$, where $n = 0, 1, 2, \ldots$, and the N_{nk} are their norms; the $P_{2n+1}(\theta)$ are Legendre polynomials. An asymptotic formula is obtained for the eigenvalues.

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EFFECTS OF THERMOCOUPLE RECESSES ON THE TEMPERATURE DISTRIBUTION IN A FUEL ROD

Yu. V. Mironov and N. S. Razina UDC 536.242

The discussion concerns a planar wall separating a layer of fissile material from a flow of heat carrier; a rectangular slot in the wall carries the thermocouple, whose head is held by a filler, whose thermal conductivity may be much less than that of the wall. The outside of the slot is coated with a metal powder whose thermophysical properties are close to those of the wall material. The internal surface of the fuel is assumed to be thermally insulated, while the heat-transfer coefficient to the liquid at the outer surface of the wall is preset, with the flow temperature near the slot assumed constant.

The problem is complex because the thermal conductivity is a discontinuous function of the coordinates. The region is divided into two zones, within which the thermal conductivity of the multilayer wall varies only along one coordinate. In each region the solution is found as an expansion in terms of generalized eigenfunctions having discontinuous derivatives at the boundaries between layers. The solutions are joined up at the interface to give an infinite system of linear algebraic equations, and the problem is closed by restricting the number of terms in the expansion.

Examples of computer calculations are presented. Tests on the convergence showed that a satisfactory agreement between the solutions, with a discrepancy of not more than 3-5% at the interface, can be obtained by retaining 20-30 terms.

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THE TRANSIENT TEMPERATURE PATTERN AROUND A NONCIRCULAR EXCAVATION

I. A. Yabko UDC 536.24

The nonstationary heat transfer between the rock and flowing air is important in calculating the heat conditions in deep pits. This is a boundary problem of the third kind for thermal conduction in a planar unbounded region bounded within by the excavation [1]. An exact solution is known for a circular tunnel [1, 2]. The paper deals with a noncircular convex shape on the assumption that the Fourier number Fo is small.

The Laplace transformation with respect to Fo reduces the problem to a regular degenerate boundary-value problem with the small parameter $\varepsilon^2 = s^{-1}$ (s is the Laplace transformation parameter) for the first derivatives; here the asymptote is given as a whole, i.e., the solution is put in the form

$$\overline{T}(M, \varepsilon) = \overline{T}_1(M, \varepsilon)[1 + \varepsilon a_1(M) + \varepsilon^2 a_2(M) + \ldots], \quad (1)$$

where M is an arbitrary point, $\overline{T}$ is the Laplace transformation of the solution to the initial problem. Expressions are derived for $\overline{T}_1(M, \varepsilon)$ and $a_1(M)$, and it is found to be possible to determine the later terms in (1).

The solution obtained by inverting (1) is used to derive the nonstationary heat-transfer factor, and an example of an elliptic tunnel is considered.

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Original article submitted January 30, 1073.

THERMAL STRESSES IN AN UNBOUNDED PLATE WITH TEMPERATURE DEPENDENT THERMAL DIFFUSIVITY

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UDC 621.78

An unbounded plate with a one-dimensional temperature distribution has normal stresses σ_{xx} and σ_{yy}, which are given by the following [3]:

$$\sigma_{xx} = \sigma_{yy} = \frac{1}{1-\mu}\left(\alpha E T_p + \frac{1}{2R}\int_{-R}^{R} \alpha E T_p\, dx + \frac{12x}{8R^3}\int_{-R}^{R} \alpha E T_p x\, dx\right). \quad (5)$$

The temperature distribution $T_p = T(x, t) - T_0$ as a function of the coordinate x and time t may be found for the case where the thermal diffusivity a is dependent on temperature via logical transformations of a standard equation [1] for the temperature of a homogeneous unbounded plate uniformly heated at the start and with a linear surface temperature variation. A semiempirical equation has been obtained for T_p in the following form on the assumption that da/dT is small and a/a_0 can be put as a linear function of T:

$$T_p = \frac{bR^2}{a_0}\,\frac{\mathrm{Fo} - \frac{1}{2}\left[1-\left(\frac{x}{R}\right)^2\right]}{1 - kb\,\mathrm{Fo}\left[1-\left(\frac{x}{R}\right)^2\right]}. \quad (4)$$

The symbols here are as in [1], with k the coefficient in the linear interpretation of a/a_0.

If the rate of change of temperature b is constant, integration of (5) with (4) gives an expression for the stress distribution over the cross section in dimensionless form:

$$\sigma_{xx} = \sigma_{yy} = \frac{1+2L\,\mathrm{Fo}}{4L\sqrt{(1+L)L}}\ln\left(1+2L+2\sqrt{(1+L)L}\right) - \frac{1+2L\,\mathrm{Fo}}{2L\left\{1+L\left[1-\left(\frac{x}{R}\right)^2\right]\right\}}, \quad (6)$$

where $L = k|b|\mathrm{Fo}$.

Checks on the calculations showed that (4) describes the temperature distribution to 10-15% for Fo > 1 and b < 2000 deg/h (the distribution was calculated with a Luk'yanov hydrointegrator); the stresses calculated from (6) were found to be 20-25% too high (Fo ≈ 2) relative to those found numerically.

For practical purposes, the accuracy given for the temperature by (4) and for the stresses by (6) is quite sufficient.

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Original article submitted May 11, 1973.

SIZES OF STEAM BUBBLES PRODUCED BY SMALL PORES

I. I. Markov

UDC 536.423

Theoretical and experimental studies have been made on the detachment diameter D_0 for a vapor bubble as a function of pore radius R when the liquid wets the heated surface.

Bubble growth in small pores is discussed on the assumption that the center of gravity of a bubble is at height h/2; the ends of the base of a bubble are fixed throughout the bubble growth up to detachment and lie at x = R.

The problem is solved by variational methods from the condition for a minimum in the total energy, which equals the sum of the surface energy

$$U_1 = \pi R^2(\sigma_{20} - \sigma_{12}) + 2\sigma_{10}\pi \int_0^h x\sqrt{1+(x')^2}\,dy \tag{1}$$

and the potential energy

$$U_2 = -\frac{1}{2} g(\rho' - \rho'')\pi \int_0^h hx^2 dy. \tag{2}$$

The problem is that of determining the minimum in the functional

$$U = \pi R^2(\sigma_{20} - \sigma_{12}) + 2\pi\sigma_{10} \int_0^h \left(x\sqrt{1+(x')^2} - \frac{x^2 h}{4a^2} \right) dy. \tag{3}$$

The Euler-Lagrange equation for the conditional minimum is

$$\frac{d}{dy}\frac{xx'}{\sqrt{1+(x')^2}} - \sqrt{1-(x')^2} + \frac{xh}{2a^2} - \lambda = 0. \tag{4}$$

The boundary conditions give the first integral in (4) as

$$\frac{x}{\sqrt{1+(x')^2}} = \frac{x^2 h}{4a^2} - \lambda x. \tag{5}$$

We put $x' = \cot\alpha$ to get

$$\sin\alpha = \frac{xh}{4a^2} - \lambda. \tag{6}$$

As

$$\frac{dx}{dy} = \cot\alpha, \text{ then } dy = \frac{dx}{\cot\alpha}. \tag{7}$$

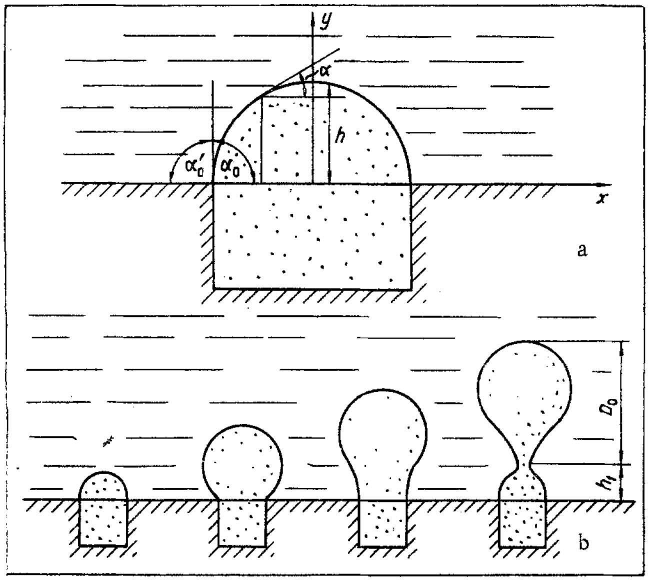

Fig. 1. a) Shape of bubble at a pore at the start; b) variation in shape and in α_0 and α_0' during growth.

We differentiate (6) with respect to x and α to get

$$\cos\alpha\, d\alpha = \frac{h}{4a^2} dx, \tag{8}$$

and so

$$dx = \frac{4a^2}{h}\cos\alpha d\alpha. \tag{9}$$

We substitute (9) into (7) to express y as

$$y = \int \frac{4a^2}{h}\sin\alpha d\alpha = -\frac{4a^2}{h}\cos\alpha + C_2, \tag{10}$$

from which

$$\cos\alpha = -\frac{h(y - C_2)}{4a^2}. \tag{11}$$

We eliminate α from (6) and (11) and determine λ from (6) with x = R on the basis that $C_2 = 0$, which is given by (10) with y = 0 and α = 90° (Fig. 1), and for x = 0 and y = h we get

$$h^3 + R^2h - 8a^2R = 0. \tag{12}$$

The real root of this equation is

$$h = \sqrt[3]{8a^2R}\ . \tag{13}$$

The height of the neck in these bubbles is very small, so

$$D_0 = \sqrt[3]{8a^2R}\ . \tag{14}$$

This formula agrees well with experiment.

NOTATION

U is the energy; h is the bubble height; D_0 is the breakaway diameter; ρ' and ρ" are the derivatives of liquid and vapor; g is the acceleration due to gravity; a is capillary constant; σ is the surface tension; and λ is the undetermined Lagrange multiplier.

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AN ENERGY MODEL FOR THE DISPLACEMENT OF LIQUID BY VAPOR IN THE CIRCULATION LOOP OF AN EVAPORATOR

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UDC 532.529.5+621.187.16.072

The column theory [1, 2] is usually employed in calculations on the motion of liquid by vapor (gas) in vertical industrial tubes; however, it has correctly been pointed out [3] that the model does not correspond adequately to the actual process. Unfortunately, no studies on this topic have yielded convenient calculation methods.

Here we propose an energy model for the flow of a fluid in such tubes; this model indicates that the flow occurs as a result of the energy provided by the bubble piston moving in the same direction as the liquid. It is assumed that the bubble moves faster than the liquid (the phases are in relative motion).

This model gives equations whose solution defines the useful work done by the bubble piston; this work is as follows for the motion of a vapor-liquid mixture in unheated vertical tubes:

$$L = \int_0^W \frac{\varphi}{1-\varphi} \frac{r^2}{CT} \frac{\rho''}{\rho'} \, dW. \tag{1}$$

The vapor content has to be known in order to use this formula for practical purposes; published methods of determining this are complex and difficult to apply. The value has been determined by minimizing the total energy of the mass flow, and this gives the true vapor content as

$$\varphi = \frac{\eta w_0''}{\eta w_0'' + w_0'}, \tag{2}$$

where $\eta = (\rho''/\rho')^{1/3}$.

Results are given from experimental tests on (2).

Substitution of (2) into (1) allows one to calculate this useful work for a variety of equipment; in particular, the energy model has been applied to evaporators of various styles with separate boiling zones. The equation for the circulation speed in the loop then gives the following expression:

$$w_c = \sqrt[4]{\frac{W^2 r^2 \eta}{f^2 T c \rho'^2 \left\{ \Sigma_h \xi + \frac{1}{n^2} \left[\left(j + \frac{W\eta}{f w_c \rho''} \right)^2 + \frac{W}{f w_c \rho''} \right] \right\}}}. \tag{3}$$

Test results are given for evaporators of various sizes from pilot plants with heating surfaces of 5 m^2 to full-scale industrial plants with 2700 m^2 with wide variation of the technological parameters. For instance, the temperature was varied from 45 to 115°C, the hydraulic resistance $\Sigma_h \xi$ from 5 to 40, and so on, and the discrepancy between theory and experiment for the speed did not exceed 10%.

NOTATION

w_0'' and w_0' are the reduced vapor and liquid velocities in lift tube; ρ'' and ρ' are the densities of vapor and liquid; T is the absolute temperature of secondary steam;

n is the ratio of cross sections of lift tube and heating tubes; φ is the true bulk steam content; W is the amount of steam formed in apparatus; r is the latent heat of evaporation; c is the specific heat of solution; $\Sigma_h\xi$ is the total hydraulic-resistance coefficient of nonboiling part of loop; f is the cross section of heating tubes; w_c is the circulation speed (speed in heating tubes).

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Original article submitted July 17, 1972.

ELECTROLYTE SCALE DEPOSITION MECHANISM FOR BOILING ELECTROLYTE SOLUTIONS

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UDC 536.423.1

The scaling is determined by interaction between the solids, the liquid, and the heat-transfer surface.

If ionic adsorption occurs on microscopic areas the rules for selective adsorption apply, only oppositely charged ions will be adsorbed; ions of opposite sign retained by electrostatic attraction near the adsorbed ions will form a double electrical layer near the surface.

The metal surface has an anode-cathode structure, so the double layer is inhomogeneous; the first layer is deposited in accordance with the laws of selective adsorption, while the second at the anode is formed via the most stable bonds (minimal solubility), while that at the cathode is due to the maximal chemical activity of the elements.

In electrolyte solutions, the forces between the ions and the heating surface are sufficient to retain the double-layer ions when a steam bubble is formed, so the growing bubble merely displaces the ions radially from its center, and at the edge of the maximum contact spot one gets a certain supersaturation. Spontaneous crystallization occurs at the vapor-solid-liquid phase interface, and this deposits minute crystals on the heating surface, which form a ring of scale in the shape of the bubble.

The scale rings are thicker than the double layer and do not persist long if the solution is far from saturated. New rings are formed while previously formed ones dissolve in unsaturated solutions. Either process may predominate in accordance with the steam formation rate and solution concentration.

When scale is deposited generally, channels are formed in the deposit, in which the solution can circulate; any material too large to pass through the microscopic circulation loops is deposited in the pores to form supersaturated complexes, where it crystallizes. This consolidates the scale and increases its thermal conductivity.

The final crystalline scale has a fairly dense and strong structure, with individual elements taking the form of fine filaments of crystalline material, which penetrate into the numerous minute cracks in the metal surface. This mode of contact between the metal and the scale prevents the latter from separating spontaneously, and is the cause of the great difficulty in removing scale from heating surfaces.

The strength of these crystalline filaments is dependent on their chemical nature, so scales differ in strength of adhesion to the metal, and also in rate of formation, the latter being determined by solution composition, surface character, and hydrodynamic and other conditions.

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Original article submitted March 16, 1973.

SOLUTION OF SOME STEADY-STATE PROBLEMS IN THE THEORY OF THERMAL CONDUCTION

V. T. Ivanov and F. V. Lubyshev UDC 536.24

Steady-state problems in the theory of thermal conduction result in integration of Poisson's equations with heat sources disposed on lines and surfaces; sometimes, the strengths of these sources are not known, but one knows the temperature of the total amount of heat from the source. A method of handling such problems is presented, which is based on solution of inverse problems for differential equations.

The following problems are considered:

$$\text{I.}\ \frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=-\frac{\omega(y)}{k}\delta(x-x_0)\ (0<x<a,\ 0<y<b), \tag{1}$$

$$(\alpha_1 u-\beta_1 u_x')_{x=0}=\varphi_1(y),\quad (\alpha_2 u+\beta_2 u_x')_{x=a}=\varphi_2(y), \tag{2}$$

$$(\alpha_3 u-\beta_3 u_y')_{y=0}=\psi_1(x),\quad (\alpha_4 u+\beta_4 u_y')_{y=b}=\psi_2(x), \tag{3}$$

$$u(y,\ x_0)=f(y)\ (c\leqslant y\leqslant d). \tag{4}$$

$$\text{II.}\ \frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}=-\frac{\omega(z)}{2\pi kr}\delta(r-r_0)\ (R_1<r<R_2,\ 0<z<H), \tag{5}$$

$$(\alpha_1 u-\beta_1 u_r')_{r=R_1}=\varphi_1(z),\ (\alpha_2 u+\beta_2 u_r')_{r=R_2}=\varphi_2(z), \tag{6}$$

$$(\alpha_3 u-\beta_3 u_z')_{z=0}=\psi_1(r),\quad (\alpha_4 u+\beta_4 u_z')_{z=H}=\psi_2(r), \tag{7}$$

$$u(z,\ r_0)=f(z)\ (c\leqslant z\leqslant d). \tag{8}$$

$$\text{III.}\ \frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \varphi^2}=-\frac{\omega(\varphi)}{kr}\delta(r-r_0)\ (R_1<r<R_2,\ 0\leqslant\varphi<2\pi), \tag{9}$$

$$(\alpha_1 u-\beta_1 u_r')_{r=R_1}=\varphi_1(\varphi),\quad (\alpha_2 u+\beta_2 u_r')_{r=R_2}=\varphi_2(\varphi), \tag{10}$$

$$u(r,\ \varphi)=u(r,\ \varphi+2\pi),\ u(r_0,\ \varphi)=f(\varphi)\ (c\leqslant y\leqslant d), \tag{11}$$

where α_i and β_i are nonnegative constants, $\alpha_i+\beta_i>0$ $(i=\overline{1,\ 4})$, $\varphi_i(y)$, $\varphi_i(z)$, $\varphi_i(\varphi)$, $\psi_i(x)$, $\psi_i(z)$ $(i=1,\ 2)$ are known functions, and the strengths of the sources $\omega(y)$, $\omega(z)$, $\omega(\varphi)$ are unknown and are to be determined; δ is a Dirac delta-function, and k is the thermal conductivity.

If the temperature distribution along the internal source is unknown, but the total amount of heat from the source is known, and it is also known that the temperature is constant, then the following condition can be added to (1)-(3) in problem I:

$$\int_c^d |u'_x(x_0-0, y)|\,dy + \int_c^d |u'_x(x_0+0, y)|\,dy = \frac{Q}{k}, \quad u(x_0, y) = T_0, \quad (c \leqslant y \leqslant d),$$

where Q is the amount of heat received from unit length of source.

Solutions to problems I-III have been obtained by differential-difference methods and integral Fourier, Hankel, and Mellin transformations. The method of solution can be extended to regions composed of rectangles. It is clearly possible to consider an arbitrary finite number of sources arranged along the line $x = x_0, x_1, \ldots, x_m$. It is possible to optimize the distributions by solving the inverse problems in conjunction with mathematical programming. An extension is made to equations with variable coefficients. It is shown that the solution to the inverse problem converges. A numerical experiment is reported.

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DISTRIBUTION OF THE STATISTICAL CHARACTERISTICS OF THE TEMPERATURE OVER THE THICKNESS OF A HEAT-TRANSMITTING WALL

V. A. Vorob'ev and O. V. Remizov

UDC 517.63:621.1

A direct-flow steam-generating tube has wall temperature fluctuations, which are of random character, and which may arise from various causes. The causes of the pulsations in the economizer and the superheater zone are turbulent effects. In the evaporator zone, i.e., from the start of surface boiling to the start of deteriorating heat transfer, the wall-temperature fluctuations are due to the random formation and collapse of clumps of steam bubbles. At the start of the zone of deteriorating heat transfer, the pulsations are due to the varying contact of the surface with liquid or steam.

When measurements are made on temperature fluctuations at channel walls, the thermocouples often lie at a certain distance from the surface and the purpose of the study is to establish the damping of the temperature pulsations in the wall thickness.

A method is presented for calculating the statistical characteristics of the temperature fluctuations (mathematical expectation, spectral density, and dispersion) for an infinite plate. Laplace transformation is used with integral expansion of the random process.

The following two cases are considered:

1) The variation in temperature pulsation through the wall thickness, with an internal heat source, with a temperature pulsating on one side and the other side thermally insulated. The transfer function is found as

$$\Phi(i\omega, x) = \frac{\operatorname{ch}\sqrt{\frac{i\omega}{a}}\,x}{\operatorname{ch}\sqrt{\frac{i\omega}{a}}\,\delta};$$

2) the temperature fluctuation varies through the wall thickness with the heat-transfer factor fluctuating on one side and a fixed heat-transfer factor on the other.

The transfer function in this case is

$$\Phi(i\omega, x) = \frac{\frac{\lambda}{\alpha}\sqrt{\frac{i\omega}{a}}\operatorname{ch}\sqrt{\frac{i\omega}{a}}\,x + \operatorname{sh}\sqrt{\frac{i\omega}{a}}\,x}{\frac{\lambda}{\alpha}\sqrt{\frac{i\omega}{a}}\operatorname{ch}\sqrt{\frac{i\omega}{a}}\,\delta + \operatorname{sh}\sqrt{\frac{i\omega}{a}}\,\delta}.$$

NOTATION

ω is the circular frequency; x is the current coordinate; a, λ, and α are the thermal diffusivity, thermal conductivity, and heat-transfer coefficient; δ is the wall thickness.

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THERMODYNAMIC PROCESSES OF A REAL GAS

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Reversible processes of a real gas in systems of variable volume V, mass G, and specific entropy s are examined on the basis of differential coefficients of deviation [1], the ratios of the respective p-v-T derivatives of real and ideal gases. The derivation of the equations of adiabatic processes with different exponents — temperature, volume, and volume-temperature exponents — is shown on the basis of different expressions of the law of conservation of energy (for the internal energy and the enthalpy) and of the equation of state of a real gas in the form pv = zRT. A graph is presented showing the variation in the adiabatic exponents of nitrogen with the initial parameters p = 10 bar and T = 270°K along the adiabat up to a pressure of 1000 bar.

In the derivation of the equation for the polytrope an expression for the total pressure differential is used:

$$dp = (\partial p/\partial V)_{G,s}\, dV + (\partial p/\partial G)_{V,s}\, dG + (\partial p/\partial s)_{V,G}\, ds. \tag{1}$$

The values of the partial derivatives are determined from the equations for the corresponding partial processes: adiabatic with G = const and adiabatic with V = const from the adiabatic equation with a temperature exponent $\varkappa$, and isochronous with V, G = const. As a result, we obtain the equation

$$\frac{dp}{p} + \varkappa\frac{dV}{V} - \varkappa\frac{dG}{G} - \varkappa\frac{ds}{c_p} - \varkappa\frac{dz}{z} = 0, \tag{2}$$

which represents the differential form of the equation of state relating the generalized potential with all the generalized coordinates [2]. To obtain the equation for the polytrope we substitute into it the parameters [3]

$$\gamma = -(dG/G)/(dV/V), \quad \vartheta = -(ds/c_p)/(dV/V),$$

connecting the variations in G and s in the process with the variations in V:

$$\frac{dp}{p} + \varkappa(1 + \gamma + \vartheta)\frac{dV}{V} - \varkappa\frac{dz}{z} = 0. \tag{3}$$

The constancy of the ratio of the exponent of the process to the exponent of the adiabat is taken as the definition of a polytropic process of a real gas, and with this condition Eq. (3) is integrated by parts [4]. The resulting equation for the polytrope contains an exponential multiplier which depends on the course of variation of the exponent along the polytrope. An equation for the polytrope connecting the temperature with the volume is obtained in this way.

In the calculation of processes using the equations obtained the properties of real gases which enter into these equations — adiabatic indices, heat capacities, compressibility factors, etc. — can be calculated with the help of tables of thermodynamic functions [5].

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EQUATION OF STATE FOR MULTICOMPONENT GAS MIXTURES

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UDC 536.7

On the basis of the concepts of the compartment method for dense gases a new equation of state is obtained for gas mixtures consisting of q components:

$$pV = RT\left(1 + \frac{\sum_{i=1}^{q} n_i B_i}{V} + \frac{\sum_{i=1}^{q} n_i C_i}{V^2} + \dots\right).$$

Here B_i and C_i are the second and third virial coefficients of the i-th fictitious substance, for which the constants of the Lennard-Jones 6-12 potential function are determined as follows:

$$\varepsilon_i = \left(\sum_{j=1}^{q} n_j \varepsilon_{ij} \sigma_{ij}^6\right)^2 \Big/ \left(\sum_{j=1}^{q} n_j \varepsilon_{ij} \sigma_{ij}^{12}\right),$$

$$\sigma_i^3 = \sqrt{\left(\sum_{j=1}^{q} n_j \varepsilon_{ij} \sigma_{ij}^{12}\right) \Big/ \left(\sum_{j=1}^{q} n_j \varepsilon_{ij} \sigma_{ij}^6\right)},$$

$$\varepsilon_{ij} = \sqrt{\varepsilon_{ii}\varepsilon_{jj}}, \quad \sigma_{ij} = \frac{1}{2}(\sigma_{ii} + \sigma_{jj}),$$

where ε_{ii}, σ_{ii}, ε_{jj}, σ_{jj} are the respective constants for the i-th and j-th components of the mixture.

Expressions for the thermodynamic properties of gas mixtures can be found using known thermodynamic equations and the equation of state obtained.

Substantiation is provided for the method developed.

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MODEL OF TOTAL VISCOSITY IN THE BOUNDARY REGION OF A TURBULENT BOUNDARY LAYER

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UDC 532.13

An equation for the total viscosity, which can be recommended for the calculation of flow in the boundary layer, including regions of laminar, transitional, and turbulent modes, is proposed in the report.

It is suggested that the total ("effective") viscosity can be represented in the form of a linear combination of the molecular and molar (turbulent) viscosities:

$$\mu_{\Sigma}=k_1\mu+k_2\varepsilon,$$

where k_1 and k_2 are free coefficients. Obviously, with greater distance from the wall the total viscosity coefficient must change to the turbulent viscosity coefficient ε (the value of ε is henceforth expressed in accordance with the Prandtl equation), while upon approach to the wall the viscosity coefficient must degenerate to the molecular viscosity μ. These conditions will be satisfied if, in particular, one takes

$$k_1=\left[1+\beta\left(\frac{\eta}{\eta_*}\right)^m\right]^{-1},\quad k_2=\beta\left(\frac{\eta}{\eta_*}\right)^m\left[1+\beta\left(\frac{\eta}{\eta_*}\right)^m\right]^{-1},\quad \eta=\frac{yv^*}{\nu}.$$

The values of β, m, and η_* are assumed to be independent of the coordinate η. We determine the values of m and β from experimental data, for which we examine the simplest case of the flow of an incompressible gas at a flat nonpermeable plate. In this case the value of η_*, which characterizes the transition from the laminar to the turbulent mode of flow, is taken as $\eta_* = 11.64$ at $Re \approx 10^6$. Integrating the equation of motion in the boundary region using the total viscosity, we obtain a velocity profile in which we determine the parameters $m = 2$ and $\beta = k^2 = 0.16$ from the condition of best conformity with experiment (the data of Reichard, Nikuradse, and Kim).

The universality of the constants m and β obtained can be demonstrated through comparison of calculated values with experimental data for more complicated flows. Calculations were made for a certain velocity profile with blowing (suction) at the wall and with a longitudinal pressure gradient. The results are in fully satisfactory agreement with the experimental data of Moffat, Keys, and Newman.

It is interesting to note that the additional viscosity from turbulent mixing decreases upon approach to the wall as a function of the fourth power of the coordinate, which corresponds to the experimental data of Daisler and Hanratty.

The calculations conducted and their comparison with experimental data indicate the universality of the proposed equation of total viscosity and the universality of the constants $m = 2$ and $\beta = k^2 = 0.16$ entering into it. It is noteworthy that no new empirical constants appeared here; β is expressed through the well-known constant $k = 0.4$, and $m = 2$ corresponds to the exponent in the Prandtl equation for the turbulent viscosity.

Using the proposed total viscosity equation an expression for the friction τ can be written in convenient explicit form (solved for the friction)

$$\tau=\mu_{\Sigma}\frac{\partial u}{\partial y},\quad \mu_{\Sigma}=\mu\varphi(A,\ \eta_*),$$

$$\varphi(A,\ \eta_*)=\left[A^2-\eta_*^2+\sqrt{(\eta_*^2-A^2)^2+4\eta_*^2A}\right](2A)^{-1},\quad A=\frac{\rho l^2}{\mu}\frac{\partial u}{\partial y},$$

and l is the mixing length.

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EXPERIMENTAL STUDY OF AN UNDEREXPANDED JET ESCAPING FROM A NOZZLE WITH A SLANTED CUT

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UDC 532.522

Experimental studies were conducted on an air jet which emerged from conical nozzles with aperture half-angles $\alpha = 1.5$, 3, and 6°. The planes of the exit cross sections of the nozzles were skewed relative to the axis of symmetry at angles $\Psi = 40$-90° (the latter angle corresponds to a straight nozzle). Two types of nozzles with Mach numbers of 1.6 and 2.5 at the sharp rim of the exit cross section were used, and the nonrated factor of the discharge, equal to the ratio of the pressure at the sharp rim to the pressure in the medium, was varied in the range from 2.5 to 20.

Shadow photography of the jet and measurement of the total pressure with a Pitot tube mounted within a cylindrical shield, which made it possible to obtain an error not exceeding 1% in the total pressure measurements at skew angles of ±40° for the oncoming stream [1], were carried out in the course of the experiments. The pressure pickup was shifted steadily perpendicular to the nozzle axis using a special coordinating device.

As a result of the experiments it was established that the jet is turned toward the cutoff part of the nozzle, with the total pressure profiles becoming equalized with greater distance from the exit cross section of the nozzle and then becoming symmetrical not only relative to the plane of symmetry dividing the jet into two equal parts but also relative to some straight line lying in the plane of symmetry and inclined to the nozzle axis by the angle $\Delta\Psi$, which is taken as the turning angle of the jet axis. The turned axis of the jet and the nozzle axis intersect near the point of interaction of the prime characteristics converging from the nozzle rims. The experimental value of the angle $\Delta\Psi$ is considerably smaller than the calculated angle determined on the basis of the system presented in [2].

An analysis of oscillograms of the total pressure and of the shadow photographs indicates the total symmetry of the shape of the prime body of the jet in the plane perpendicular to the plane of symmetry and containing the nozzle axis. In addition, the geometrical dimensions of the jet coincide with the dimensions of an axially symmetrical jet with a Mach number at the exit cross section equal to the Mach number of the slant-cut nozzle in this plane.

This makes it possible to use calculated functions for an ordinary axially symmetrical jet [3] to calculate the flow geometry in the indicated plane in the presence of a nozzle with slanted cut.

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DYNAMICS OF A TWO-PHASE MEDIUM AT LOW ACCELERATIONS

D. P. Alekseev

UDC 532.529.6

It is shown that empirical relationships for the speed of gas bubbles and resistance vectors under various conditions applicable to terrestrial gravitation can be extrapolated to the region with very much weaker mass forces. The empirical relationships are transformed into identical dimensionless ones, with Reynolds, Frood, and Bond numbers as independent variables.

The mode of rise and speed w can be determined for an unbounded liquid and for a liquid bounded by a cavity for certain ranges in the mass forces j and characteristic linear dimensions R of the gas bubble.

If the size of the gas bubble is comparable with that of the cavity, there is a marked effect on the behavior from the deformation of the phase interface. Relationships are given in inexplicit form between the bubble deformation parameters and the Bond number Bo for bubbles in the form of pulsating and stable spheroids. The deformation of a mushroom-shaped bubble is also discussed.

Experiments have also been performed with weak mass forces using evaporation in a freely falling container. It is concluded that the empirical relationships derived for terrestrial gravitation need only be slightly corrected to describe satisfactorily the behavior of a two-phase medium for j of 0.05-0.25 m/sec^2 (Fig. 1c). Some numerical examples are given to illustrate the corrected empirical relationship.

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A HORIZONTAL STABILIZED HETEROGENEOUS FLOW IN PNEUMATIC TRANSPORT

M. E. Dozin

UDC 532.529.5

An experimental study has been made of the relation between the resistance factor for horizontal transport line and the similarity criteria for a stabilized disperse flow transporting powder or granular materials.

The differential equations for such a flow go with the conditions of uniqueness [1] to give the dimensionless equation

$$\lambda - \lambda_0 = \Phi\left(\mu,\ Fr_s,\ \frac{\rho_0}{\rho_s},\ \frac{D}{d},\ \frac{k_e}{D}\right), \tag{1}$$

where λ and λ_0 are resistance coefficients for the transport and the gas flow, μ is the concentration by weight, Fr_s if the Frood number for the solid component, ρ_0 and ρ_s are the densities of the carrying medium and solid component, and D and d are the diameters of tube and particle; k_e is the hydraulic roughness of the tube.

The main experiments were performed on a pneumatic transport system with horizontal straight pipes of length 33 m. The pipes were completely free from bends, which considerably reduced the pressure fluctuations in the measuring section and improved the accuracy of pressure-difference measurement. The design of the equipment provided for a wide range in parameter variation and automatic recording.

Each series of the main runs was performed so that one of the numbers in (1) was varied widely to determine directly its effects on the function, with all the other numbers constant; the results gave a working relationship for the additional resistance in the form

$$\lambda - \lambda_0 = A\mu \frac{\rho_s}{\rho_0} \left(\frac{D}{d}\right)^{0.7} \mathrm{Fr}_s^{-0.8} \left(\frac{k_e}{D}\right)^n. \quad (2)$$

The general character of (2) was confirmed for pneumatic transport of various powders and granular materials; A and n were found to take the following values: granular materials in polished tubes gave $A = 0.35 \cdot 10^{-3}$ and $n = 0$, while powders gave $A = 0.25$ and $n = 0.7$.

Processing of these data and published ones indicate that the error in determining the resistance factor from (2) does not exceed 11%, except in isolated cases. It is shown to be necessary to incorporate the tube roughness in generalizing the experimental data.

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DROPLET SIZE SPECTRUM DETERMINATION BY PULSE COUNTING

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Pulse counting is a good method of droplet size determination, particularly for polydisperse, high-speed, and high-temperature gas-liquid flows [1]. This has been utilized in the thermal power laboratory at Lenin Polytechnical Institute, Kharkov [2]. The measurement concerns the frequency h(S) with which droplets short-circuit electrodes, and this gives the unnormalized density distribution $f(D)$ for the droplet sizes by solving the integral equation

$$h(S) = \frac{1}{2} \int_S^\infty \left[D^2 \arccos \frac{S}{D} - S (D^2 - S^2)^{\frac{1}{2}}\right] f(D)\, dD, \quad (1)$$

where D is the drop diameter and S is the distance between electrode ends.

It is found that Wicks and Dakler solved (1) inaccurately, and the solution is unstable under change in the step and choice in the upper limit of integration D_m.

An exact analytical solution has therefore been found for (1).

We differentiate (1) twice and transfer to new variables to obtain an integral equation of Abel type having an exact analytic solution, which can be put in terms of the previous variables as

$$f(D) = \frac{1}{\pi D} \int_D^\infty (S^2 - D^2)^{\frac{1}{2}} h^{\mathrm{IV}}(S)\, dS. \quad (2)$$

In many cases, the observed h(S) can be approximated closely by

$$h(S) = B \exp(-\alpha S). \quad (3)$$

In that case,

$$f(D) = \frac{\alpha^3 B}{\pi} K_1(\alpha D), \quad (4)$$

where $K_1(\alpha D)$ is a Bessel function. Then the normalized density distribution for the diameter takes the form

$$v(D) = \frac{2\alpha^4}{3\pi} D^3 K_1(\alpha D). \tag{5}$$

If $f(D)$ is found numerically from (2), the integration is carried up to a finite limit.

The distribution found from the exact solution to the integral equation eliminates the major errors arising from the numerical method used by Wicks and Dakler. Experimental test shows that the method can be realized at transsonic speeds in a two-phase medium at pressures up to 40 bar at high temperatures.

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